Exam Quantum Physics 2

Date	16 June 2015
Room	A. Jacobshal 02
Time	9:00 - 12:00
Lecturer	D. Boer

- Write your name and student number on every separate sheet of paper
- Raise your hand for more paper
- You are allowed to use the book "Introduction to Quantum Mechanics" by Griffiths
- You are *not* allowed to use print-outs, notes or other books
- The weights of the exercises are given below
- Answers may be given in Dutch
- Illegible handwriting will be graded as incorrect
- Good luck!

Weighting

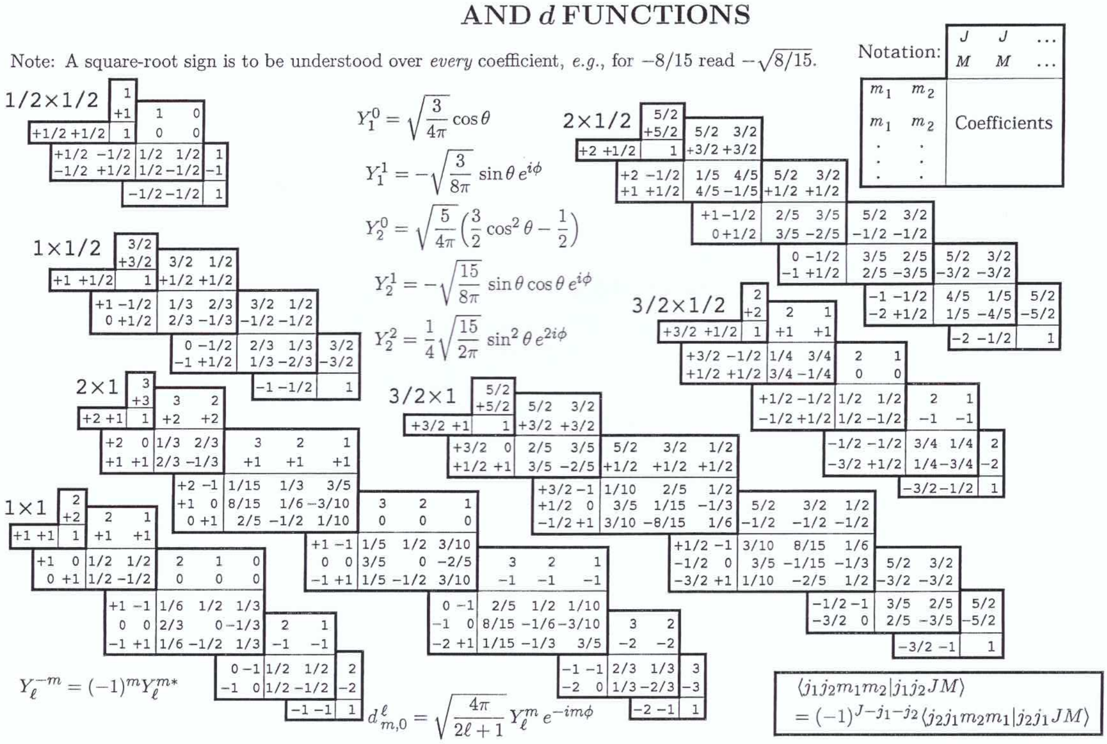
1a)	6	2a)	5	3a)	6	4)	12
1b)	12	2b)	10	3b)	14		
1c)	10	2c)	10	3c)	5		

$$\text{Result} = \frac{\sum \text{points}}{10} + 1$$

Exercise 1

(a) In the hyperfine structure of ^{35}Cl one encounters electron and nuclear spin quantum numbers $s = \frac{1}{2}$ and $i = \frac{3}{2}$, respectively. Denote the quantum numbers of $\boldsymbol{F}^2$ and F_z by f and m_f, where $\boldsymbol{F} = \boldsymbol{S} + \boldsymbol{I}$ is the total spin. Use the table below to write down the Clebsch-Gordan decompositions of the total spin states $|s, i; f, m_f\rangle = |\frac{1}{2}, \frac{3}{2}; 1, 1\rangle$ and $|s, i; f, m_f\rangle = |\frac{1}{2}, \frac{3}{2}; 1, 0\rangle$ and relate these two decompositions by using a raising or lowering operator.

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS



(b) Calculate the commutators of the angular momentum operators L_i $(i = 1, 2, 3)$ and a Hamiltonian with a central potential V, i.e. $[L_i, -\frac{\hbar^2 \vec{\nabla}^2}{2m} + V(|\vec{r}\,|)]$. Use the results to show that the energy levels of an electron experiencing a central potential do not depend on the quantum number m.

(c) Consider the effect of a uniform electric field along the $\hat{z}$ direction on the four $n = 2$ levels of hydrogen. Recall that the Stark effect is described by the term $H_S = eEz$ in the Hamiltonian. Indicate which matrix elements $\langle 2\, l'\, m' | H_S | 2\, l\, m\rangle$ vanish and explain why.

Exercise 2

Consider a two-dimensional square well potential:

$$V(x,y) = \begin{cases} 0 & \text{for } 0 \le x \le a \text{ and } 0 \le y \le a \\ \infty & \text{elsewhere} \end{cases}$$

(a) The first excited state of the system is degenerate. Give its energy and the explicit expressions for the corresponding wave functions.

Next introduce the perturbation:

$$H'(x,y) = \begin{cases} V_0 & \text{for } 0 \le x \le a/2 \text{ and } 0 \le y \le a/2 \text{ and } V_0 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(b) When using degenerate perturbation theory for the first excited states one generally has to consider off-diagonal matrix elements of H'. Explain using symmetry arguments how in the case of this specific perturbation H', one can avoid considering off-diagonal elements.

(c) Calculate in first order perturbation theory the correction(s) to the energy level of the first excited state and indicate for which potentials V_0 the result is expected to be valid. You may make use of the following integral:

$$\int_0^{a/2} \sin(n\pi x/a)\sin(n\pi x/a)dx = \frac{a}{4}, \quad \int_0^{a/2} \sin(\pi x/a)\sin(2\pi x/a)dx = \frac{2a}{3\pi}.$$

Exercise 3

Consider the one-dimensional potential:

$$V(x) = \begin{cases} c\,x & \text{for } x \geq 0 \\ \infty & \text{elsewhere} \end{cases}$$

with positive constant c.

(a) Explain which of the following trial wave functions (all with positive real parameter b, normalization A, and vanishing for $x < 0$) would be acceptable to determine an upper bound on the ground state energy:

$$A\frac{e^{-bx}}{x},\ Ae^{-bx},\ Axe^{-bx},\ A\sin(bx),\ A\frac{x}{x^2+b^2},\ A\frac{x^2}{x^2+b^2}.$$

(b) Determine the best approximation to the energy of the ground state that one can achieve with the variational method using the following trial wave functions for real parameter b:

$$\psi_T(x) = \begin{cases} A\,x\,e^{-bx} & \text{for } x \geq 0 \\ 0 & \text{elsewhere} \end{cases}.$$

You may make use of the following integrals:

$$\int_0^\infty x^n e^{-bx} dx = \frac{n!}{b^{n+1}}.$$

(c) Can one find a trial wave function that would give an upper bound on the first excited state? Motivate your answer.

Exercise 4

Consider the Hamiltonian $H = H^0 + H'(r,t)$, where H^0 is time independent and H' is a time-dependent perturbation. Consider the particular case of a two-level system consisting of states ψ_1 and ψ_2 with unperturbed energies $E_1^{(0)}$ and $E_2^{(0)}$. Let $H'(r,t) = V(r)\sin(\omega t)\theta(t)$, for real potential V with non-vanishing $V_{ij} \equiv \langle\psi_i|V(r)|\psi_j\rangle$ for all i,j. In the rotating wave approximation the probability to be in state 2 for $t \geq 0$, if the system is in state 1 for $t < 0$, is given by

$$P_2(t) = \frac{|V_{21}|^2}{\hbar^2\omega_R^2}\sin^2\left(\frac{\omega_R t}{2}\right)$$

with

$$\omega_R \equiv \left[(\omega_{21}-\omega)^2 + \frac{|V_{21}|^2}{\hbar^2}\right]^{\frac{1}{2}} \quad \text{and} \quad \omega_{21} = \frac{E_2^{(0)} - E_1^{(0)}}{\hbar}.$$

Explain what is the rotating wave approximation and provide the conditions on ω and t for which the approximation is expected to be valid. Show that the maximum probability to be in state 2 is independent of V.